Testing the Fisher effect under fractional integration

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Abstract. In this paper we model inflation and the nominal interest rate as a fractionally integrated, autoregressive, moving average (ARFIMA) process in order to test the theoretical proposition that nominal interest rates move one for one with inflation, thus, leaving the real interest rates unchanged; i.e. the Fisher effect. Using the necessary integration conditions first derived by Fisher and Seater (1993) and extended to fractional orders by Jensen et al. (forthcoming), we test the Fisher effect hypothesis in eleven developed countries; Belgium, Canada, Denmark, France, Germany, Greece, Ireland, Japan, the Netherlands, the UK, and the US. For each country, we first estimate and test the order of integration of inflation and the nominal interest rate with a univariate ARFIMA model. In seven of the eleven countries, the estimated relative order of integration between the inflation and interest rate series directly rules out a Fisherian link between inflation and nominal interest rates. One country's order of integration fails to identify the Fisher effect and hence the hypothesis cannot be tested (Ireland). Because of the estimated orders of integration, testing for the Fisher effect in the three remaining countries (Belgium, Greece, and the US) requires the novel approach of applying the identification scheme of King and Watson (1997) to the fractionally differenced series. As in previous studies, we find little support for the Fisher effect.

Keywords: Fisher Effect, Fractional Integration, Long-Memory, Long-Run Neutrality
JEL Classification: C2; E4

1 Introduction

The Fisher effect is a long-run neutrality (LRN) proposition that requires nominal interest rates to move one for one with inflation so that the real interest rate is unaffected. Past empirical tests of the Fisher equation have restricted the orders of integration of the relevant series to be from the set of integer numbers. They then employed unit-root and/or mean-trend stationary models to establish the theoretical conditions necessary for determining whether the Fisherian link holds (see King and Watson, 1997; Koustas and Serletis, 1999).[1] There is, however, a more general model whose order of integration is a continuum of real numbers. Called a fractionally integrated model, this model not only nests the unit-root and trend-stationary processes within it, but the model also include processes exhibiting more complex medium- and persistent long-run dynamics.

Jensen et al. (forthcoming) has extended the Fisher and Seater (1993) necessary order of integration restrictions for testing LRN to a bivariate fractionally integrated, autoregressive,

[1] For a literature review on the broader area of testing for long-run neutrality see Bullard (1999).

moving average model (ARFIMA). Using Sowell (1992) exact maximum likelihood estimator of the ARFIMA model to estimate and test the order of integration in each of the eleven country's nominal interest rate and inflation series, we find the Fisher effect does not hold in seven countries independent of the identification method of the structural parameters discussed by King and Watson (1997). To test for the Fisher effect in Belgium, Greece, and the US, we rely on the identification method of King and Watson (1997), and again find little evidence in support of the Fisher effect for these countries.

The remainder of the paper is organized as follows. Sections 2 summarizes Jensen et al. (forthcoming) order conditions necessary for testing the Fisher effect. These conditions are then applied to empirical data in Section 3 for the eleven countries. In Section 4, the King and Watson (1997) identification approach to testing the Fisher effect is applied to the fractionally integrated series of Belgium, Greece, and the US. We summarize our empirical findings in Section 5.

2 Integration Conditions

In Table 1, we list Jensen et al.'s (forthcoming) necessary order of integration conditions, rewritten in terms of the Fisher effect hypothesis. Let $d_\pi \in I\!R$ denote the order of integration of inflation, $d_R \in I\!R$ the order of integration of nominal interest rates, and $\gamma_{R\pi}$ the long-run derivative of nominal interest rate to a change in inflation. In Table 1, L is the lag operator, $L^j x(t) \equiv x(t-j)$, $j = 0, 1, 2, \ldots$, and $(1-L)^d$ is the fractional differencing operator defined by the binomial expansion:

$$(1-L)^d = \sum_{j=0}^{\infty} \pi_j L^j, \text{ where } \pi_j = \prod_{0<k\leq j} \frac{k-1-d}{k}, \quad j = 0, 1, 2, \ldots$$

The fourth column of Table 1 lists the implications each case has on the long-run derivative of nominal interest rate to inflation.[2] In Section 3, we will find that of the five cases in Table 1, we primarily see Case (*ii*), Case (*iii*), and Case (*iv*), with five of the eleven countries exhibiting Case (*iv*), three countries Case (*iii*), and two countries Case (*ii*). The only other was Case (*i*) for Ireland.

Except for the fractional nature of the order of integration, Case (*iii*) is the same necessary condition used in previous tests of the Fisher effect (King and Watson, 1997; Koustas and Serletis, 1999). Under Case (*iii*), an exogenous inflation shock will have a permanent affect on both inflation and the nominal interest rate. Hence, for this case testing the

[2] For the derivation of each case see Jensen et al. (forthcoming).

Case	Relative Order of Integration	Economic Meaning	Fisher Effect
i)	$0 < d_\pi < 1$	No permanent stochastic changes to π.	Unidentified
ii)	$0 < d_R < 1 \leq d_\pi$	Permanent stochastic changes to π, no permanent stochastic changes to R.	Reject
iii)	$1 \leq d_\pi = d_R$	Permanent stochastic changes to π and R.	$\gamma_{R\pi}$
iv)	$1 \leq d_R < d_\pi$	Permanent stochastic changes to $(1-L)^{d_\pi - 1}\pi$, no permanent stochastic changes to $(1-L)^{d_\pi - 1}R$.	Reject
v)	$1 \leq d_\pi < d_R$	No permanent stochastic changes to $(1-L)^{d_R - 1}\pi$.	Unidentified

Table 1: The Fisher effect hypothesis' integration conditions

Fisher hypothesis requires estimating the long-run derivative between the nominal interest rate and inflation, $\gamma_{R\pi}$, and testing that it is equal to one.

Cases (*ii*) and (*iv*) are very similar in their interpretation of the Fisher effect hypothesis. In both cases, because $1 \leq d_\pi$ inflation will be permanently affected by an exogenous shock. However, this shock will not have a permanent affect on the nominal interest rate. Because in Case (*ii*) $d_R < 1$, the nominal interest rate is mean reverting with dynamics that converge to the behavior found before the shock to inflation. Thus, the nominal interest rates are only temporarily affected by the permanent change in inflation.

Case (*iv*) can be understood in a similar manner. By differencing both inflation and the nominal interest rate $d_\pi - 1$ times, the differenced inflation series will be a unit-root process, whereas, the differenced nominal interest rate series is integrated of order less than one. A shock to inflation will permanently affect the differenced inflation series, but will have only a temporary effect on the differenced nominal interest rate. This relationship between the differenced series also holds between inflation and the nominal interest rate, thus, the Fisher effect hypothesis is rejected.

3 Fractional integration properties of the data

All of the data utilized comes from the International Financial Series database. Each series is measured quarterly and includes a short term nominal interest rate, R, and the change in logged consumer price index, π, for each of the eleven countries listed in the first column of Table 2. All the series begin between 1957:1 and 1972:1 and end in 2003:2.

To determine which of the Cases in Table 1 these eleven countries satisfy, we employ Sowell's (1992) exact maximum likelihood approach and estimate an array of ARFIMA(p,d,q) models:

$$\Phi(L)(1-L)^d(x(t)-\mu)=\Theta(L)\epsilon(t), \tag{1}$$

where $-1 < d < 1/2$, $\Phi(L) = 1-\phi_1 L-\phi_2 L^2-\cdots-\phi_p L^p$ and $\Theta(L) = 1+\theta_1 L+\theta_2 L^2+\cdots+\theta_q L^q$ are polynomials of p and q degrees, respectively, with roots outside the unit circle, $\epsilon(t) \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_\epsilon^2)$, and μ is the unknown mean of $x(t)$, for $p,q = 0,1,2,3$, and $\Delta\pi$ and ΔR.[3]

Table 2 reports the parameter estimates (standard deviations in parenthesis) and the value of the log-likelihood function, $\mathcal{L}$, for each country's best Schwarz (1978) information criterion (SIC) selected ARFIMA model of inflation and interest rate growth rates.[4] We also compute the Q-statistics associated with the estimated models and find no remaining structure in the residuals. Thus, the estimated ARFIMA models of Table 2 appear to fit the data quite well.

Even though the long-run effect of a unit shock is zero for a fractionally integrated series whose $d < 1$, the energy found near the origin in the process's spectrum suggests that the effect of the shock could last for years. If this persistence constitutes a large percentage of the data then the number of degrees of freedom is small, making any inference about the long-run effect of a shock difficult. In a similar vain, the effect of a shock to a unit-root process will never dissipate. Thus, to accurately model unit-root dynamics requires an infinite number of observations, but an economist will never have the observations needed to make meaningful inferences about the series' infinite behavior. So the ARFIMA and unit root models are observationally equivalent when the ARFIMA model has an impulse response function whose cycle is the entire length of the observable data. Under these circumstances, both models produce the same conclusion about the existence of the Fisher effect.

[3]In simulations Cheung and Diebold (1994) found that subtracting out the sample mean prior to estimation with the exact maximum likelihood estimator of an ARFIMA model (feasible exact MLE) produced superior estimates when compared to Fox and Taqqu (1986) frequency-domain estimator. Hence, we apply the feasible exact MLE to each of the eleven country's inflation and nominal interest rate series.

[4]Because the growth rate of a series, x, equals $(1-L)x$, the order of integration for the level series is, $d_x = 1 + d_{\Delta x}$. Hence, the order of integration for inflation and the short term nominal interest rate equals one plus the differencing parameter estimates found in Table 2.

Country Time Period	Series	d	ϕ_1	ϕ_2	θ_1	θ_2	$\mathcal{L}$	Power
Belgium								
(1957.1-2003.3)	ΔR	-0.0507	0.3585				226.9439	0.325
		(0.1433)	(0.1659)					
(1957.1-2003.2)	$\Delta\pi$	0.0920					427.5506	0.19
		(0.0561)						
Canada								
(1957.1-2003.3)	ΔR	-0.1727			0.4214*		96.9800	0.561
		(0.0792)			(0.0787)			
(1957.1-2003.2)	$\Delta\pi$	0.4646*					700.8133	
		(0.0344)						
Denmark								
(1972.1-2003.3)	ΔR	-0.0413					32.0815	0.168
		(0.0867)						
(1957.1-2003.2)	$\Delta\pi$	0.4314*	-0.3191*				594.6088	
		(0.0541)	(0.0838)					
France								
(1957.1-1999.1)	ΔR	-0.2702	0.4976				119.2290	0.549
		(0.1945)	(0.2082)					
(1957.1-2003.2)	$\Delta\pi$	0.3304*	-0.8634*		1.3553*	0.5822*	666.1478	
		(0.0585)	(0.0789)		(0.0766)	(0.0798)		
Germany								
(1957.1-2003.3)	ΔR	-0.5356*	0.7628*				99.6297	0.725
		(0.1304)	(0.1009)					
(1957.1-2003.2)	$\Delta\pi$	0.3333*	0.0111	-0.9914*	-0.0260	0.9224*	707.9605	0.980
		(0.0567)	(0.0129)	(0.0168)	(0.0537)	(0.0688)		
Greece								
(1961.1-2003.3)	ΔR	0.2439*					230.4957	0.887
		(0.0592)						
(1957.1-2003.2)	$\Delta\pi$	0.2801*	-0.9912*		0.9254*		405.6626	0.772
		(0.0509)	(0.0096)		(0.0317)			

Country Time Period	Series	d	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	$\mathcal{L}$	Power
Ireland										
(1962.1-2003.3)	ΔR	0.1083				0.5411			29.9312	0.059
		(0.0934)				(0.0823)				
(1957.1-2003.2)	$\Delta\pi$	-0.3413*	1.0785*	-0.7710*	0.6557*	-0.4788*	0.5334*		568.4145	0.497
		(0.1187)	(0.1239)	(0.1685)	(0.1240)	(0.1700)	(0.1348)			
Japan										
(1957.1-2003.3)	ΔR	-0.1019	-0.9789*			1.5180*	0.6909*		-31.1968	0.454
		(0.0661)	(0.0213)			(0.0735)	(0.0623)			
(1957.1-2003.2)	$\Delta\pi$	0.4044*	-0.9960*			0.9637*			598.7757	
		(0.0481)	(0.0057)			(0.0196)				
Netherlands										
(1960.1-1998.4)	ΔR	-0.2812*							-58.2701	0.996
		(0.0740)								
(1957.1-2003.2)	$\Delta\pi$	0.0429							395.9134	0.058
		(0.0569)								
U.K.										
(1957.1-2003.3)	ΔR	-0.3088	0.5514*						143.9503	0.588
		(0.1409)	(0.1367)							
(1957.1-2003.2)	$\Delta\pi$	0.4790*	-0.9814*	-0.9765*	-0.9944*	0.9642*	0.9286*	0.9311*	613.2791	
		(0.0254)	(0.0230)	(0.0248)	(0.0091)	(0.0889)	(0.0960)	(0.0415)		
U.S.										
(1957.1-2003.3)	ΔR	-0.1201				0.6537*			125.6569	0.394
		(0.0654)				(0.0572)				
(1957.1-2003.2)	$\Delta\pi$	0.1933	0.8152*			-0.3106	-0.3578*	0.4483*	764.5960	
		(0.2927)	(0.1769)			(0.1419)	(0.0725)	(0.0903)		

Table 2: Feasible exact maximum likelihood estimates of the best SIC selected ARFIMA(p, d, q) models where $p, q = 0, 1, 2, 3$, with standard deviations contained in parenthesis and the value of the log-likelihood function, $\mathcal{L}$. $*$ denotes significance at the 1% level. The Power column reports the empirical power of the Wald-stat for the hypothesis, $H_0 : d = 0$.

To identify between an ARFIMA model estimated with too few degrees of freedom and a unit-root process's infinite persistence, we generate 1000 restricted ($d = 0$) and unrestricted ARFIMA(p, d, q) series for each model in Table 2 and use the unrestricted series to determine the empirical power of the relevant test statistics and the restricted series to produce the empirical size of the test statistics. The results from the restricted data reveals that the empirical size of the test-statistics are nearly identical to their theoretical size, and hence, to conserve space we choose not to report them. In the last column of Table 2 is the empirical power of the Wald-stat for the null hypothesis $H_0 : d = 0$ using the 5% critical value.

In Table 3, we group together the eleven countries by the value of their estimated d_π. Group A consists of those countries (Ireland) with $d_\pi < 1$ and hence, fail to satisfy the permanent change to inflation necessary for testing the Fisher effect. The inflation rate for the countries (Belgium, Netherlands, and the US) comprising Group B follow a unit root process ($d_\pi = 1$) and can thus be tested for the Fisher effect. Lastly, Group C are those countries (Canada, Denmark, France, Germany, Greece, Japan, and the UK) whose inflation rate is integrated of order greater than one ($d_\pi > 1$).

Out of the eleven countries only Ireland fails to satisfy the condition that inflation be permanently affected by a shock; i.e. $d_\pi \geq 1$. However, strong persistence is evident in the estimated model of the growth rate of Ireland's CPI since its autoregressive polynomial, $1 - 1.0785L + 0.771L^2 - 0.6557L^3$, has a root close to one. In the second best model of Ireland's inflation process as determined by the SIC:

$$(1 + \underbrace{0.0655}_{0.1008} L - \underbrace{0.8142}_{0.1130} L^2)(1 - L)^{0.4726}\Delta\pi_t = (1 - \underbrace{0.2574}_{0.1687} L + \underbrace{0.6725}_{0.1189} L^2)\epsilon_t$$

with $\hat{\sigma}^2 = 0.0001$, $\mathcal{L} = 565.167$ and the parameter estimate's standard error are found below the curly braces ($s.e.(d_{\Delta\pi}) = 0.0356$), the near unit root behavior in the autoregressive parameters of Table 2 is lost but the necessary persistence dynamic is statistically evident in the significant $d_{\Delta\pi} = 0.4726$.[5] The above long-memory parameter, $d_{\Delta\pi} = 0.4726$, is significantly greater than Ireland's $d_{\Delta R} = 0.1083$, and hence, by Case (*iv*) the Fisher effect is rejected.

Of the remaining ten countries, the difference between d_π and d_R in seven (Canada, Denmark, France, Germany, Japan, Netherlands, and the UK) of these countries is also large enough to directly reject the Fisher effect and conclude that a permanent change to

[5]Of the array of ARFIMA models estimated for Ireland's inflation rate only the ARFIMA(3,d,1) model of Table 2 and the ARFIMA(1,d,0) model produced negative estimates of $d_{\Delta\pi}$ along with autoregressive polynomials with roots close to one. All the other estimated ARFIMA model's $d_{\Delta\pi}$ were significantly greater than zero and significantly greater than $d_{\Delta R}$.

Fisher effect test results for Group A ($d_\pi < 1$)

Country	d_R	*Case*	Fisher Effect
Ireland	$d_R = 1$	*i*	Unidentified

Fisher effect test results for Group B ($d_\pi = 1$)

Country	d_R	*Case*	Fisher Effect
Belgium	$d_R = d_\pi = 1$	*iii*	$\gamma_{R\pi}$
Netherlands	$d_R < 1$	*ii*	–
U.S.	$d_R = d_\pi = 1$	*iii*	$\gamma_{R\pi}$

Fisher effect test results for Group C ($d_\pi > 1$)

Country	d_R	*Case*	Fisher Effect
Canada	$d_R = 1$	*iv*	–
Denmark	$d_R = 1$	*iv*	–
France	$d_R = 1$	*iv*	–
Germany	$d_R < 1$	*ii*	–
Greece	$d_R = d_\pi$	*iii*	$\gamma_{R\pi}$
Ireland*	$d_R = 1$	*iv*	–
Japan	$d_R = 1$	*iv*	–
U.K.	$d_R = 1$	*iv*	–

*Ireland's second best BIC model of π.

Table 3: Fisher effect hypothesis for Group A countries (those countries whose inflation series are fractionally integrated of order less than one), Group B countries (those countries where a unit-root in their inflation series cannot be rejected) and Group C countries (those countries whose inflation series are fractionally integrated of order greater than one).

inflation does not permanently affect the short-term nominal interest rate (Case (*ii*) and (*iv*)).

The order of integration in the three remaining countries (Belgium, Greece, and the US) are not significantly different from each other, requiring us to estimate the long-run derivative $\gamma_{R\pi}$ to determine if the Fisher effect holds (Case (*iii*)). For Greece this difference is evident in $d_{\Delta\pi} - d_{\Delta R} = 0.0362$. For Belgium and the US we conducted likelihood ratio tests where the restricted model sets the long-memory parameter equal to the estimated value from the other series' model.

The restricted interest rate model for Belgium where $H_0 : d_{\Delta R} = d_{\Delta\pi} = 0.0920$ is tested is:

$$(1 + \underbrace{0.2166}_{0.0721} L)^{0.0920} \Delta R_t = \epsilon_t$$

with $\hat{\sigma}^2 = 0.0051$, $\mathcal{L} = 226.401$. This produces a LR-stat of 0.0004 with a p-value of 0.984 and thus, we cannot reject that inflation and nominal interest rates in Belgium are integrated of the same order.[6]

The estimated restricted US interest rate model where $d_{\Delta R} = d_{\Delta\pi} = 0.1933$:

$$(1 - L)^{0.1933} \Delta R_t = (1 + \underbrace{0.5228}_{0.0751} L)\epsilon_t$$

with $\hat{\sigma}^2 = 0.0165$, $\mathcal{L} = 117.167$, gives a LR-stat of 0.14 whose p-value is 0.708. In addition, the restricted US inflation model with $d_{\Delta\pi} = d_{\Delta R} = -0.1201$:

$$(1 + \underbrace{0.9352}_{0.0283} L)^{-0.1201} \Delta\pi_t = (1 - \underbrace{0.1310}_{0.0746} L - \underbrace{0.3376}_{0.0647} L^2 + \underbrace{0.3838}_{0.0754} L^3)\epsilon_t$$

with $\hat{\sigma}^2 = 0.00001$, $\mathcal{L} = 764.217$, produces a LR-stat of 0.001 and a p-value of 0.975. Thus, both LR-stats fail to reject the null that the order of integration in US inflation and nominal interest rate series are equal.

4 The Fisher effect hypothesis in the Belgium, Greece, and the US

To estimate the long-run derivative for Belgium, Greece, and the US, we extend the methodology of King and Watson (1997) to fractionally differenced series. Consider the following bivariate vector autoregressive model of order p for fractionally differenced inflation,

[6] Since Belgium's best fitting model of $\Delta\pi$ is an ARFIMA(0,d,0) model we are only able to test the hypothesis, $H_0 :\ d_{\Delta R} = d_{\Delta\pi} = 0.0920$.

$(1-L)^{d_\pi}\pi_t$, and fractionally differenced nominal interest rate, $(1-L)^{d_R}R_t$:

$$\begin{aligned}(1-L)^{d_\pi}\pi_t &= \lambda_{\pi R}(1-L)^{d_R}R_t + \sum_{j=1}^{p}\alpha_{\pi R}^{j}(1-L)^{d_R}R_{t-j} \\ &\quad + \sum_{j=1}^{p}\alpha_{\pi\pi}^{j}(1-L)^{d_\pi}\pi_{t-j} + \epsilon_t^{\pi} \qquad (2)\\ (1-L)^{d_R}R_t &= \lambda_{R\pi}(1-L)^{d_\pi}\pi_t + \sum_{j=1}^{p}\alpha_{RR}^{j}(1-L)^{d_R}R_{t-j} \\ &\quad + \sum_{j=1}^{p}\alpha_{R\pi}^{j}(1-L)^{d_\pi}\pi_{t-j} + \epsilon_t^{R} \qquad (3)\end{aligned}$$

where $\lambda_{\pi R}$ and $\lambda_{R\pi}$ are respectively the contemporaneous effects of nominal interest rates on inflation and inflation on nominal interest rates, and ϵ_t^{π} and ϵ_t^{R} are random shocks to inflation and nominal interest rates, respectively. In matrix notation, Eq. (2)-(3) can be written as:

$$\alpha(L)X_t = \epsilon_t, \qquad (4)$$

where:

$$\alpha(L) = \sum_{j=0}^{p}\alpha_j L^j, \qquad X_t = \begin{bmatrix}(1-L)^{d_\pi}\pi_t \\ (1-L)^{d_R}R_t\end{bmatrix}, \qquad \epsilon_t = \begin{bmatrix}\epsilon_t^{\pi} \\ \epsilon_t^{R}\end{bmatrix},$$

$$\alpha_0 = \begin{bmatrix}1 & -\lambda_{\pi R} \\ -\lambda_{R\pi} & 1\end{bmatrix}, \qquad \alpha_j = -\begin{bmatrix}\alpha_{\pi\pi}^{j} & \alpha_{\pi R}^{j} \\ \alpha_{R\pi}^{j} & \alpha_{RR}^{j}\end{bmatrix}, \quad j = 1, 2, ..., p.$$

The long-run multipliers $\gamma_{R\pi}$ and $\gamma_{\pi R}$ measure the long-run response of nominal interest rates to inflation and the long-run response of inflation to nominal interest rates, respectively, and are defined in terms of the VAR parameters as, $\gamma_{R\pi} = \alpha_{R\pi}(1)/\alpha_{RR}(1)$ and $\gamma_{\pi R} = \alpha_{\pi R}(1)/\alpha_{\pi\pi}(1)$. The Fisher effect holds if $\gamma_{R\pi} = 1$

Since π_t and R_t are treated as endogenous, as was pointed out by King and Watson (1997), Eq. (4) is econometrically unidentified. Identification requires an a priori restriction on either $\lambda_{\pi R}$, $\lambda_{R\pi}$, $\gamma_{\pi R}$, or $\gamma_{R\pi}$, as well as assuming that the exogenous shocks, ϵ_t^{π}, and ϵ_t^{R}, are independent. Rather than estimate the model using a single parameter restriction we follow King and Watson (1997) and estimate the above model using a range of values for the three parameters. We estimate the model using the same method outlined in King and Watson (1997) except that we first difference the data using the estimated d_π and d_R found in Table 2. Since we are only interested in $\gamma_{R\pi}$, we allow $\lambda_{\pi R}$, $\lambda_{R\pi}$ and, $\gamma_{\pi R}$ to take on a reasonable range of values and graphically report in Figure 1 through 3 the estimates of $\gamma_{R\pi}$ along with their 95% confidence intervals over the values of these identifying conditions.

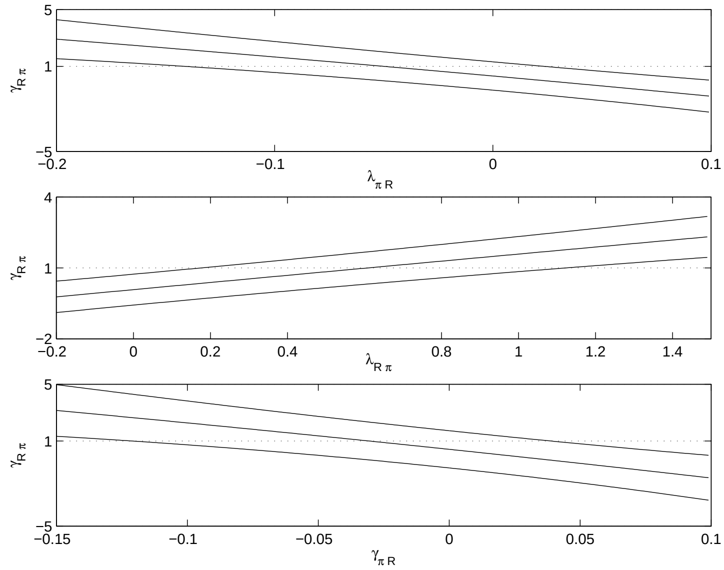

Figure 1: Fisher effect for Belgium; (top) 95% confidence interval for $\gamma_{R\pi}$ as a function of $\lambda_{\pi R}$, (middle) 95% confidence interval for $\gamma_{R\pi}$ as a function of $\lambda_{R\pi}$, (bottom) 95% confidence interval for $\gamma_{R\pi}$ as a function of $\gamma_{\pi R}$.

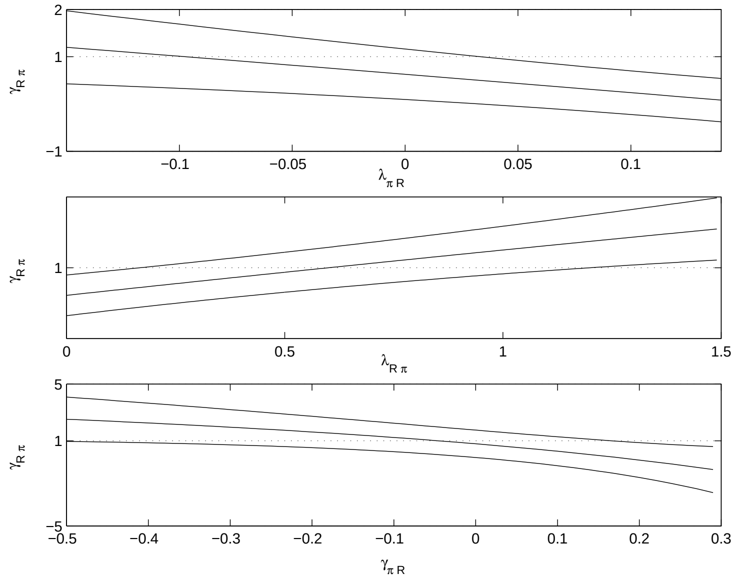

Figure 2: Fisher effect for Greece; (top) 95% confidence interval for $\gamma_{R\pi}$ as a function of $\lambda_{\pi R}$, (middle) 95% confidence interval for $\gamma_{R\pi}$ as a function of $\lambda_{R\pi}$, (bottom) 95% confidence interval for $\gamma_{R\pi}$ as a function of $\gamma_{\pi R}$.

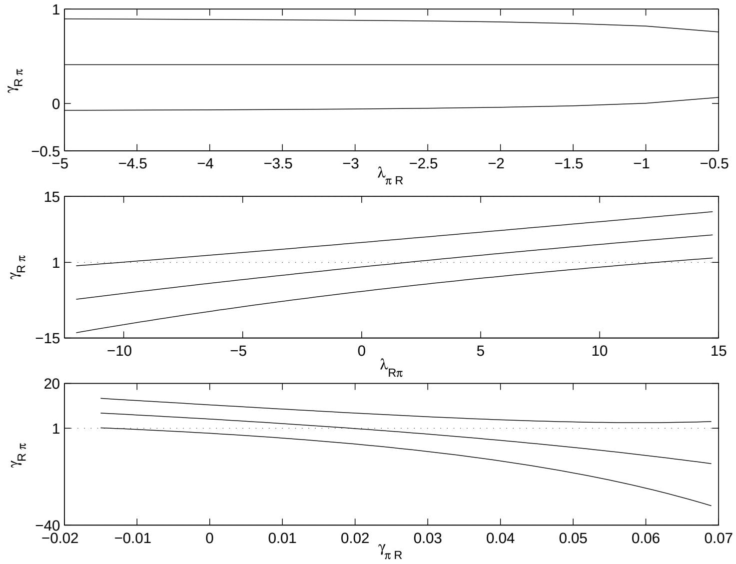

Figure 3: Fisher effect for the US; (top) 95% confidence interval for $\gamma_{R\pi}$ as a function of $\lambda_{\pi R}$, (middle) 95% confidence interval for $\gamma_{R\pi}$ as a function of $\lambda_{R\pi}$, (bottom) 95% confidence interval for $\gamma_{R\pi}$ as a function of $\gamma_{\pi R}$.

Figures 1 through 3 suggest that there is subjective evidence both for and against the long-run Fisher relation. Unlike Koustas and Serletis (1999), there is some hope that the Fisher effect exists if one assumes that the contemporaneous relationship between nominal interest rates and inflation is positive ($\lambda_{\pi R} > 0$). But these values of $\lambda_{\pi R}$ are very small and only occur for Belgium and Greece (see the top panel of Fig. 1 and 2). We can accept the Fisher effect in all three countries when a positive long-run impact of nominal interest rates on inflation ($\gamma_{\pi R} > 0$) is assumed. However, this again only occurs for very small values of $\gamma_{\pi R}$ (see the bottom panel of Fig. 1-3). The strongest evidence of a Fisherian link between inflation and nominal interest rates is found when the contemporaneous relationship between inflation and nominal interest rates is positive and greater than 0.5 ($\lambda_{R\pi} \geq 0.5$). Under this identification scheme the Fisher effect hypothesis cannot be rejected in either of the three countries.

From the above evidence for Belgium, Greece, and the US, it is difficult to draw any substantial conclusion about any one of the three identifying parameter's effects on $\gamma_{R\pi}$. Some discernible patterns are the negative relationship between $\gamma_{R\pi}$ and the contemporaneous and long-run effects of nominal interest rates on inflation ($\lambda_{\pi R}$ and $\gamma_{\pi R}$), along with the strong positive relationship between $\gamma_{R\pi}$ and the contemporaneous effect of inflation on nominal interest rates, $\lambda_{R\pi}$. Other than these three stylized facts we are unable to find a significant thread between the three countries and leave our results to the subjective interpretation of others.

5 Conclusion

Classical economic theory states that real economic variables should not be affected by changes to nominal variables. Hence, the Fisher relation states that changes in prices should not have an effect on the real interest rate, so it follows that the nominal interest rate should move one for one with inflation. The vast majority of empirical studies, however, reject the Fisher relation (see King and Watson, 1997 and Koustas and Serletis, 1999). Typically nominal interest rates are found to move less than one for one with inflation.[7] Using a fractionally integrated model we find inflation having no long-run effect on nominal interest rates in seven of the eleven countries tested; these being Canada, Denmark, France, Germany, Japan, the Netherlands, and the United Kingdom. Ireland's inflation series failed to satisfy the necessary order condition and thus, could not be tested for the Fisher effect.

We tested Belgium, Greece, and the US using the methodology of King and Watson (1997) by first fractionally differencing the series with their estimated order of integration. Still we are only able to find a small amount of evidence supporting the Fisher effect. Therefore, we conclude that allowing the order of integration of the data series to take on noninteger values does not reconcile the professions inability to identify the Fisher relation empirically. Thus, future theoretical and empirical research is necessary to solve this puzzle.

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[7] For a counterexample see Mishkin (1984), who found a strong effect for Canada, the UK, and the US.

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